Section 8.2 Quadratic Formula

Given: $ax^2 + bx + c = 0$ (Note ***must*** equal zero!)

Completing the square yields:

$$x^2 + \frac{b}{a}x + \qquad = -\frac{c}{a}$$

$$x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 = -\frac{c}{a} + \frac{b^2}{4a^2}$$

$$\left(x + \left(\frac{b}{2a}\right)\right)^2 = \frac{-4ac + b^2}{4a^2}$$

$$\sqrt{\left(x + \left(\frac{b}{2a}\right)\right)^2} = \sqrt{\frac{-4ac + b^2}{4a^2}}$$

$$x + \left(\frac{b}{2a}\right) = \frac{\pm\sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

We will look as problems using the Quadratic Formula:

$x^2 + 6x + 9 = 2$ We first force it to = 0

$x^2 + 6x + 7 = 0$

Since we will be using the Quadratic Formula, we will write the formula ***each*** time we use it and we will list the coefficients!

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \qquad \begin{aligned} a &= 1 \\ b &= 6 \\ c &= 7 \end{aligned}$$

Now...

$$x = \frac{-6 \pm \sqrt{36 - 28}}{2}$$

$$x = \frac{-6 \pm \sqrt{8}}{2}$$

$$x = \frac{-6 \pm 2\sqrt{2}}{2}$$

$$x = \frac{\cancel{2}\left(-3 \pm \sqrt{2}\right)}{\cancel{2}}$$

$$x = -3 \pm \sqrt{2}$$

Example $x^2 + 6x + 4 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \qquad \begin{aligned} a &= 1 \\ b &= 6 \\ c &= 4 \end{aligned}$$

$$x = \frac{-6 \pm \sqrt{36 - 20}}{2}$$

$$x = \frac{-6 \pm \sqrt{20}}{2}$$

$$x = \frac{-6 \pm 2\sqrt{5}}{2}$$

$$x = -3 \pm \sqrt{5}$$

Example $3x^2 + 7x - 2 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \qquad \begin{aligned} a &= 3 \\ b &= 7 \\ c &= -2 \end{aligned}$$

$$x = \frac{-7 \pm \sqrt{49 + 24}}{6}$$

$$x = \frac{-7 \pm \sqrt{73}}{6}$$

The text devotes section 8.3 to "Studying Solutions of Quadratic Equations".

Basically it says that a quadratic function (one that opens up or down) does one of 3 things with respect to the *x*-axis.

Page 518 Middle

Shows the 3 possibilities. Namely, the curve crosses the *x*-axis twice , only just touches the *x*-axis, or does not touch or cross the *x*-axis.

We can know which situation is at hand by considering the **discriminant.**

From the quadratic formula, $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ we have $x = \dfrac{-b}{2a} \pm \dfrac{\sqrt{b^2 - 4ac}}{2a}$.

If $b^2 - 4ac$ is positive, then we would add the square root value to get one solution and subtract the square root value to get the other value. We have two answers which represent the graph crossing the *x*-axis twice.

If $b^2 - 4ac$ is zero, then we would and subtract zero from $\dfrac{-b}{2a}$ and get the same value (of course). This represents the curve only touching the *x*-axis. We call this a "double root".

If $b^2 - 4ac$ is negative, then we have an imaginary number. While we still have two answers, since the number is complex, it represents the fact that the curve does not cross the *x*-axis.

All of this is due to the *complete quadratic formula*:

If $\mathrm{a}x^2 + bx + c = 0$ then $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

In our final case, for instance, $\mathrm{a}x^2 + bx + c \neq 0$ so we have complex roots.

Section 8.4 Applications involving quadratic equations.

Example 1, page 521 is poorly done. Do ***not*** use the method shown in the text.

The problem tells us that Fiona rode a motorcycle 300 miles at a particular speed. If she had averaged 10 mph more, the trip would have taken 1 hour less. We are to find the original speed.

Obviously this is a R T = D problem. We use the table:

	R	*T*	*D*
original speed	r		300
faster	r+10		300

As before with this type of problem, we divide the distance by the rate to fill in our table:

	R	*T*	*D*
original speed	r	$\frac{300}{r}$	300
faster	r+10	$\frac{300}{r+10}$	300

Of course the times are not the same. The time for the faster trip is less than for the slower trip so we add 1 hour to the lesser time to equal the longer time.

$$\frac{300}{r} = \frac{300}{r+10} + 1$$
$$300(r+10) = 300r + r^2 + 10r$$
$$300r + 3000 = 300r + r^2 + 10r$$
$$\cancel{300r} + 3000 = \cancel{300r} + r^2 + 10r$$
$$0 = r^2 + 10r - 3000$$

Factoring…..

10	300
20	150
40	75
50	60

$$(r-50)(r+60) = 0$$
$$r = 50 \qquad r = -60$$

–60 is a reject since a negative speed “is not in the domain”.

Fiona rode her motorcycle 50 mph.

Problem 2, page 525 has the following table:

	R	T	D
1*st* part	r		60
2nd part	r - 4		24

The *total* time is 8.

Problem 7, page 525:

	R	T	D
to	r		36
from	$r - 3$		36

Total time is 7.

Page 525 problem 8

	R	T	D
go	R+10		600
return	R		600

This gives us: $\frac{600}{R+10}+\frac{600}{R}=22$

Multiply by the LCD to get $600R+600R+6000=22R^2+220R$

Simplifying to get
$$11R^2-490R-3000=0$$
$$(11R+60)(R-50)=0$$

Clearly R is not negative so our solution is R = 50.

Problem 11, page 526

Let w = time needed by the first well
$w + 8$ = time needed by spring

The rate is the reciprocal of each of the times so we have $\frac{1}{w}$ and $\frac{1}{w+8}$ respectively.

Thus $\frac{3}{w}+\frac{3}{w+8}=1$

Notice the question asked for the spring's time so be sure you answer the question asked.

Problem 12

Let P = larger pipe (hours)
P + 3 = smaller pipe (hours)

$$\frac{4}{p} + \frac{4}{p+6} = 1$$

p will be equal to 3 so the smaller pipe would require 6 hours.

Problem 25 You are to solve for t.

$$s = v_0 t + \frac{gt^2}{2}$$

$$\frac{gt^2}{2} + v_0 t - s = 0$$ we make it equal to zero

$$\frac{g}{2}t^2 + v_0 t - s = 0$$ to emphasize that the coefficient of t^2 is $\frac{g}{2}$

so $a = \frac{g}{2}$ and $b = v_0$ and $c = -s$

thus $t = \frac{-v_0 \pm \sqrt{(v_0)^2 - 2gs}}{g}$ using the quadratic formula